Article

Theory, Culture & Society
29(4/5) 165–192

DOI: 10.1177/0263276412443568
tcs.sagepub.com

Digital Design and Topological Control

Luciana Parisi
Goldsmiths, University of London, UK

Abstract

At the turn of the 21st century, topology, the mathematical study of spatial properties that remain the same under the continuous deformation of objects, has come to invest all fields of aesthetics and culture. In particular, the algebraic topology of continuity has added to the digital realm of binary information, the on and off states of 0s and 1s, an invariant property (e.g. a continuous function), which now governs the relation between different forms of data. As this invariant function of continual transformation has entered the field of automated computation, the culture of binary digits has shifted towards a new level of calculation derived from the introduction of temporal quantities into finite sets of algorithmic instructions and parameters. This new level of topological computation, it will be argued, defines new operative procedures of control, constantly adding axioms at the limit of calculation through an invariant function that establishes a smooth or uninterrupted connectivity between distinct data. The establishment of a continual function between distinct forms of data is based on homeomorphism or topological isomorphism between data objects, of which parametricism, as the new global style for architecture and design, is a perfect example.

Keywords

mereotopology, parametric architecture, postcybernetic control, topological continuum

Topological Urbanism

At the turn of the 21st century, topology has led to a new mathematical formalization of the relation between space and time (e.g. the instantaneous communication of ubiquitous computing), perception, cognition and memory (e.g. the automation of orientation, navigation, and mapping) and between model and matter (e.g. digital design

Corresponding author:
Luciana Parisi, Goldsmiths, University of London, London SE 14 6NW, UK
Email: l.parisi@gold.ac.uk
http://www.sagepub.net/tcs/

and construction). In sum, topology has introduced relations to the programming of culture.

This article will place this topology of continuity[1] within the field of computational architecture, in particular the digital design of urban space. Software design has turned the Euclidean grid of discrete points into a morphogenetic form of relations changing over time. The computational programming of urban settings has substituted the urban plan with a topological schema of variations directed by the capacity of algorithms to evolve and to be affected by external contingencies in *real* time. For instance, urban software for modelling water in the city (from sewer systems to storm water drainage systems and water distribution systems) or the more general digital design of prototype systems that include data and models for land use (e.g. geographic information systems, GIS), transportation analysis, cost estimation, energy usage, water, noise, airflows, etc., responds to changing conditions or calculates according to a potential urban behaviour under certain circumstances. Software applications, such as navigation systems, have most commonly been described as modes of tracking movement and orientating spatial perception. The computation of topological invariants has instead opened new conditions of interactivity between software and actual behaviour, which are included in the programming of an infinite series of scenarios. Recently, Benjamin H. Bratton has argued that the iPhone and similar handheld devices are radically changing the possibilities of spatial interaction by eclipsing the physical city, now overcome by a geo-computational space in which digital objects will be able to see, hear and comment on our interactions with them (Bratton, 2009). For Bratton, urbanists and architects should stop designing new buildings and rather focus on building new software programs to improve the use of existing urban structures and systems. As the city has become the meta-data environment of digital media, digital networks have come to share data with our nervous systems, while our bodies have become one with 'the extensional networks of the living city, both controlling its machinery at a distance... and [being controlled] psychologically by that machinery in the course of our movements' (2009: 93).

The continuous smooth feedbacks between software programming and urban behaviour is here condensed within handheld devices to suggest that data are continuously animated and transformed into maps, which control and connect with the immediate and remote environment. This direct relation with ambient information, provided by software interfaces such as the iPhone, according to Bratton, points to a spatial network made not of icons but of real conditions of connection. Bratton explains that digital urbanism is not about designing a new network of connections but requires a way to capitalize upon 'the computational mechanisms that formulate the nodal and edge conditions and the interface'. This involves a 'systematising of the possibility of particular event[s]' and a 'geo-computational program… that calculates conditions of appearance' rather than scripting beforehand what can emerge at the end (2009: 94).

What topology has brought to urban design is the capacity of long-term planning to become open to revisions, updates, real-time inputs and contingencies. As the computational power of managing and calculating data has become extended to the design of urban scenarios, real-time variations have been included within software programs so as to anticipate the emergence of potential changes. The computation of urban data will be taken here as an example of an algorithmic mode of planning defined by an extended apparatus of prediction able not only to establish the condition of the present through the retrieval of past data but also, and significantly, to change these conditions according to data variations immediately retrieved from the environment. From this standpoint, topology has also meant that the cybernetic logic of control has disclosed its mechanisms of value and measure to non-quantifiable conditions so as to capture qualitative changes *before* their emergence. But these mechanisms of anticipation or of pro-programming scenarios are not simply defined by the mathematics of division and addition, and similarly do not just rely on off and on states of 0s and 1s.

What is new here is that these mechanisms now seem to rely on the topological calculation of the continuous function, an invariant property that fills the gap between binary digits. This article suggests that this continuous function now characterizes the computational design of the urban space in the form of parametric aesthetics. In particular, parametric aesthetics serves here to suggest that the topological approach to urban design is based on the introduction of qualitative variations and temporal evolution in the predictive calculation of data, which account for potential urban scenarios. This is why topology implies a transformation in strategies of control, whereby the software interaction with the real data of the environment has become constitutive of a postcybernetic logistics. Far from simply reducing biophysical variables and contingencies to sets of binary codes, which are unable to process the grey areas between sequences, the topological approach to digital design coincides with the integration of differential relations, or intensive data within the generation of spatio-temporal connections. The introduction of the invariable function in computational planning also reveals that cybernetic control now relies on the calculation of differentials and uncertainties. In the computation of urban design, this is evidenced by the use of growing algorithms or open-ended instructions that respond and adapt to the external environment, thus including contingencies into programming.

Parametric aesthetics is thus a mode of computational control relying on the capacities of algorithms to create the perception of space as a relational field of emergence. From this standpoint, parametric aesthetics also suggests that computational control has developed its own aesthetic form, which has been associated with folds, morphologies, smooth surfaces and real-time evolving structures. In other words, control as the topological computation of space has acquired a sensuous skin, turning all points, corners and lines into planes of relations, short-circuits of immediate connection or speedy paths of variations. Here there is no core, no end point and no individual response: only the continuous fluctuation of a total form enveloping all parts.

The article also discusses how topology implies an ontological concern for the mathematical formalization of the relation between finite and infinite sets. In particular, the article will briefly address the mathematical formalization of the continuum problem and the systematization of infinitesimals leading to the development of topology. The Leibnizian quest for infinitesimals, together with Deleuze's concept of differential relations, will be specifically considered as crucial to the ontological constitution of topology, resulting in a computational design based on contingent variabilities or temporalities. Parametric aesthetics, however, inherits the onto-mathematical diatribe between extension as a field of continual variations (determined by an underlying infinitesimal series) and extension as a sequence of spatio-temporal actualities able to connect and disconnect. This diatribe will be here discussed by emphasizing the contrast between topology and its aesthetics of smooth control, and mereotopology, offering us an aesthetic of discontinuous relations between control and events.

Parametric aesthetics indeed reveals that the topological mode of calculating relations, where all parts become incorporated into one evolving whole, is not exhaustive of all relations, and of the algorithmic sequential relations in particular. On the contrary, parametric aesthetics precisely involves the quantification of data as parameters, which cannot be overlooked and simply become dissolved into continual qualities. Instead, parametric aesthetics rather points at the persistence of parts and of the relations of parts to whole without parts being always already subsumed into a whole. These parts, and in this case, parametric and algorithmic quantities, are discrete entities that may enter into a relation thanks to their capacity to select not only data coming from the environment but also to predict data that is not possible to compute. This other face of parametric aesthetics will be explained through Alfred N. Whitehead's notion of mereotopology, insofar as the relation between parts and wholes is central to a study of the relation between infinite and finite entities. Whitehead's mereotopological schema rejected the Leibnizian infinitesimal series and questioned Henri Bergson's predilections for temporal continuity by arguing that what connects points are actual entities on an extensive continuum. However, the Whiteheadian case of mereotopology and its schema of discontinuous relations are not simply alternative instances set against the topological aesthetics of power, using vectorial tools as instruments of control. The mereotopological schema serves here to suggest that there is no equivalence between the topological architecture of control and spatio-temporal events. With mereotopology, in other words, control and events are not in a reciprocal presupposition: topological continuities are expressions of large assemblages able to incorporate discontinuous events into a stream of infinitesimal variations, but events are not definable by infinitesimal or temporal continuities.

From this standpoint, parametricism is an example of the operative system of control defined by the computation of infrastructural networks: the smooth architecture of continual variations changing the values of parameters by responding to real data from the environment. Here parametricism deploys how control operates as a prehensive apparatus of spatio-temporal futurities. In other words, control, as Brian Massumi (2007) has brilliantly explained, is a mechanism of anticipation, whereby the *apprehension* for unknown variables indirectly works to determine the reality of the present. If topological control works, it is because what can be anticipated corresponds to what actually has to happen, foreclosing the conditions of uncertainties into pre-set probabilities in the present.

The mereotopological schema, however, offers another understanding of parametric relations, showing how parameters can themselves be conceived as actual entities entering a nexus of spatio-temporal events, whose relations are discontinuous. The very strategy of anticipation of spatio-temporalities in digital design inversely contributes to the diffusion of unintended algorithmic actualities into computational culture. These actualities are here understood as computational events. Events, according to Whitehead, involve the capacity of any actual entity (organic or inorganic) to select and become affected by pure data-objects (or eternal objects in Whitehead's terminology), which define how the indeterminate becomes determinate in an actual entity.

Alfred N. Whitehead's mereotopological schema implies that events come first: the summation of actual entities in a nexus that has selected pure data and has brought them together for the first, unique and unrepeatable space-time. From this standpoint, the article will contrast the topological view of parametric aesthetics, which assumes that variations are to be derived from the relational or infinitesimal space of contingencies lying outside the system (which are then pre-programmed in the urban model for instance), with the mereotopological insistence that parts, quantities, discontinuities exist not only at the level of actualities, but also at the general level of formality. This means that Whitehead's mereotopological schema forces us to revisit the computational significance of formal hierarchies in relation to actual contingencies. No longer are contingencies to be conceived as being external (a mere factor of extrinsic force) to the formal schema but, as this article attempts to argue, contingencies or chances are instead internal to the logical condition of any formal processing. This means that patternless structures are internal to any logic of computation and, as a result, they define any mathematical, physical or biological organization of matter as incomplete.

The article suggests that contingencies are to be found first at the level of computational processing, because it is at this level that algorithms encounter the indeterminate conditions (patternless data) for which they can become eventful. This idea of computational contingency is based on recent findings in information theory that argue for a mathematical logic (and not the statistical notion) of randomness (i.e. lack of structure), meaning that 'something is random if it can't be compressed into a shorter description. In other words, there is no concise theory that produces it' (Chaitin, 2001: 18). Chaitin's algorithmic information theory sets incompleteness and undecidability within his axiomatic system to show that it is impossible to calculate randomness, or what he defines as the uncomputable: maximally unknowable and irreducible data. Since it is impossible to calculate the size of the smallest program, as Turing and Gödel demonstrated, Chaitin concludes that computational logic implies a program-size complexity, whereby it is the program (the software, the theory) and not just its application that shows the existence of patternless infinities at the limit of actual sets of algorithms.

From this standpoint, this article does not use the example of parametric aesthetics to claim that novelty in computation is to be derived from external factors, or, for instance, by the way a discontinuous relation between software and hardware becomes an opportunity for explaining novelty in digital urbanism. This is not what is argued here. Instead this article's argument is driven by the possibility offered by the mereotopological schema of finding the conditions for novelty in digitality in the discontinuous architecture of eternal objects – uncomputable quantities – that are or are not selected by actual entities. This forced juxtaposition of the formal level of uncomputable data with the formal schema of eternal objects is in this article another way to point out the incompleteness of computation as the very condition for novelty. The article suggests that this condition is intrinsic to computation and irreducible to any interactive relation between software and hardware. In this way, mereotopological discontinuity is not an alternative to the topological form of power, which is, as argued in the first part of this article, ontologically grounded in relational continuity. If anything, the mereotopological schema of discontinuous data can help us to reveal that the predictive machine of control, which now involves, as Brian Massumi (2007) has brilliantly explained, a pre-emptive mode of power foreclosing futurity into actualities, is not the same as the uncomputable machine of the event. The latter instead, unlike control, requires that indeterminate data become determined in the cumulative processing of non-equivalent actualities.

To put it in another way, the introduction of topological invariants in computation points to an apparatus of power operating by pre-empting change and re-programming the event before this can happen, thus flattening control and novelty (or event) onto a topological matrix of continual co-evolution. On the contrary, borrowing from Whitehead's mereotopological schema of relation, it is possible to suggest that parts cannot become a whole but rather a whole can be a part that connects to another. This is also to say that if the parametric aesthetics of topological control anticipates events in its own morphogenetic body, mereotopology reveals that events are cut-bringing novelties that characterize the becoming of the extensive continuum. Events therefore do not grant continuity between entities, but, on the contrary, are the occasions for the discontinuous becoming of continuities. This explanation, however, only helps us to describe the actual level of novelty. Actual novelty instead does not come from nowhere and does not exclusively concern the physical realm. Novelty must also be explained at the level of abstract formalism. The mereotopological schema of eternal objects and actual entities proposed by Whitehead contributes to metaphysically support what in information theory is increasingly becoming unavoidable: the presence of the uncomputable in logic. The formal reality of uncomputable random data is here taken as the condition that makes any mode of computation (analogue or digital) possible.

This condition has to be found within the computational processing of algorithms, at the formal and axiomatic level. It is here suggested that uncomputable data can reveal a strange contingency within form, chance within programming. From this standpoint, uncomputable algorithms interrupt the topological co-evolution of urban software and urban behaviour. Far from establishing continuous feedback or reversible function whereby software takes command of urban behaviour or the latter feeds back on the program, the sequential running of algorithms will instead expose an uncomputable quantity of rules for an infinite quality of behaviours, which are un-provable and un-applicable spatio-temporalities. Here control becomes as random (or patternless) as the uncomputable data it tries to compress into axioms. The uncomputable triggers contingent rules within computational design. It is this new dominance of contingency within programming that demarcates the unquantifiable reality of events and the impossibility for control to incorporate and neutralize them. In particular, digital urbanism points at computational events at once discovered and constructed by the software programming of unlived spatio-temporalities. From this standpoint, this article takes parametric aesthetics as a case in which the digital design of time and space is not only controlling (or pre-empting) the emergence of events, but is instead unleashing random events or unlived worlds in urban design.

Before the case of parametric aesthetics, the article will address the mathematico-geometric and ontological notion of relational space in digital design. This discussion will contribute to the analysis of the 5Subzero's design of the responsive environment, *Topotransegrity*. The last section draws on Alfred N. Whitehead's notion of mereotopology to explain how novelty in parametric aesthetics is to be found in the uncomputable order of relations or the infinite quantities invading digital programming.

The Invariant Function

The invariant function of continual transformation has entered the field of automated computation. It has shifted the culture of binary digits towards the calculation of temporal quantities and into finite sets of algorithmic instructions and parameters. This topological computation involves operative procedures of control, constantly adding axioms at the limit of axioms through an invariant function that establishes a smooth (uninterrupted) connectivity between distinct data. The establishment of a continual function between distinct forms of data is based on homeomorphism or topological isomorphism between places or objects, of which parametricism,[2] as the new global style for architecture and design, is a perfect example.

Parametricism is here taken as an example of algebraic topology as it understands space as a field of relations and not discontinuous points (Boyer, 1989).[3] Metric distances between points are substituted by neighbourhood proximity, which, computationally speaking, include vague or incomplete quantities (at the limit of 0s and 1s) in the calculation of probabilities. For example, the introduction of indeterminacies into the source code of parametric programming has transformed the binary logic of yes and no into the fuzzy states of the logical conditions defined by *maybe* and *perhaps*. These are not merely qualitative renderings of digital binarism, for which a certain sequence may correspond to a certain shade of colour. Fuzzy states are instead to be understood as involving new processes of quantifications. The spatial architecture of points and lines, of discrete and finite states, has been superseded by topological methods of measuring infinitesimal quantities and establishing neighbourhood proximity through the function of the constant invariant. Paradoxically, however, it will be argued, the topological culture of continual variations forecloses the potential intrusion of discontinuity, unforeseen change, in the efficient continuity of cause and effects.

From this standpoint, topological thinking as a new method of quantification of uncertain states also corresponds to an operative power of control based on topological computations (i.e. the adding of invariant functions between axioms and between formal models and material implementations). Here control works not to prevent the future but to add a link to it by using the invariant function as a protocol for uncertainties. In other words, the introduction of invariant functions in computation points out that the gap between 0s and 1s is instead a relational space composed of infinitesimal points of continuity.

In the early 1990s experimentations with computational programming had already embraced the topological turn in digital design. Architect Greg Lynn (1993: 9), for instance, famously observed that each pure element of quantity, for instance a binary algorithm, was determined in a qualitative form by neighbouring forces, the vague space around the point, which unravelled the topological complexity of the generative form. These qualitative forces were, for instance, defined by the physical stress caused by environmental forces on the genetic elements of a form. Physical forces were here equivalent to the infinitesimal points of any point, turning the degrees of separation between one form and another into gradients on a curve. For Lynn, these relational points had to be included in the generative computation of form.

The inputting of physical gradients into computation, however, did not correspond to the representation of intensive quantities (the qualities of the physical stress points between terms) reduced to 0s and 1s binary states. If Leibniz admitted that the space between undivided monads was not a void, but a full texture of micropercepts and microaffects, Lynn's topological architecture suggested that these points were included in the process of computation itself: the generative movement from one set of algorithms to another exceeded the binary function. In other words, computational abstraction surpassed the representation or simulation of space. As Kipnis argued, the architecture of Deformation showed that computational techniques stimulated investigations towards a non-representational space. Computation thus involved:

> the study of camouflage methods experimenting with computer 'morphing' programs that smoothly transform one figure into another, or employing topological meshing techniques such as splines, NURBS, etc., that join surfaces delimited by the parameters of disjoint two-dimensional figures into a smoothed solid. (2009: 112)

Lynn's neo-Baroque aesthetics of a folding architecture directly responds to the continuum problem posed by Leibniz's infinitesimal or differential calculus (Boyer, 1989: 216).[4] Leibniz used the calculus as a way to solve the question of infinity: is a line between two points another point or an infinitesimal aggregation of points (increasingly small quantities that cannot be mathematically counted)?[5]

Leibniz concluded that if a line was an aggregation of points, infinitely divisible parts, then a continuum could neither be a unity nor an aggregation of unities. In other words, continua were *not real entities* at all.

Continua were 'wholes preceding their parts' and had a purely ideal character (i.e. non-physical). For Leibniz space and time, as continua, were ideal, and anything real, such as matter, was discrete, compounded of simple unit substances or *monads*.[6] But to explain the transition from finite, discrete reality to infinitesimal, transcendental magnitudes, Leibniz resorted to the philosophical law of continuity, emphasizing the role of the ratio between differentials (differential calculus), the infinitesimal differential quantity or the curve of transition between two orders of magnitude or quantities (infinite and finite series) (Boyer, 1989: 399–407).[7]

Leibniz's 'labyrinth of the continuum' described the paradoxical condition of transcendental infinities and actual finitude: how could the infinitely divisible yet be constituted by discrete unities (Leibniz, 2001).[8]

At the core of Leibniz's topological conception of space is the differential calculus as the calculation of derivatives or differential relations, describing the infinitely small quantities between two quantities (the quantity of the ordinate x and the quantity of the abscissa y).[9] Lynn's neo-Baroque aesthetics builds on the computation of infinitesimal relations to animate digital design away from the coldness of binary codes. However, as Lynn also suggests, this topological turn is not simply part of a technical and/or aesthetic movement, but more precisely addresses the metaphysical primacy of relations and processes over points and results. Folding in architecture indeed deploys the intricacy of technicality and aesthetics with metaphysics as a way to describe the cultural tendency of an epoch. The bending and twisting of lines into complex structures that loop and auto-reflect on their irregular trajectories reveals nothing other than a sense of spatiality in computational culture.

Just as Leibniz insisted that there is a transcendental ideal order of infinitely small quantities, Deleuze conceived of infinitesimals as the differential relation that supersedes actual terms. As the terms cancel each other out, the relation remains. This is a third term, which Deleuze (2004: 217–20) identifies with the tangent of a curve, a straight line that touches a curve at only one point.[10] But the infinitesimal gap between two points was no longer governed by a transcendental infinity (determined by the principle of sufficient reason). According to Deleuze, as non-standard analysis reintroduced the infinitesimal as a non-exact numerical quantity, it also provided a new axiomatic formula of differential relations. In short, the formalization of differential relations coincided with the systematization of the intuition of continuity by means of non-standard axioms (Fletcher, 1989).

From this standpoint, the differential relation was formalized as the function of an invariant, a constant x through which the continuum between discrete entities became a mathematical expression of relational continuity itself. According to Deleuze, however, the algebraic determination of indeterminate differentials (or infinitesimal dy or dx) was not simply an axiomatic solution. On the contrary, it also meant that the differential relation could not correspond to a discrete number or finite quantity (an axiom). The finite result (the invariant x) instead could only be determined by the immanence of the relation with the infinitely small: the tendency of the differential relation to vanish but of the relation to tend towards the limit z. According to Deleuze, the integration of the differential relation resulted not in a determinate point, or discrete axiom, but involved the sequential arrangement of points generating not a straight line but a curve. This curve was a function in the neighbourhood of the given tangential point: the limit of the function. The introduction of differential relations into digital design thus exposes the integration of infinite qualities as a computational limit expressed by the curve.

Parametric Aesthetics

The computation of infinitesimal relations has come to describe not only, as Lynn would have it, the neo-Baroque aesthetics of a folding architecture, but also the postcybernetic control of the continuum itself. Topology as the ultimate mathematics of smooth space now corresponds to the aesthetic of postcybernetic control based on curvature or continual variation: differential relations have become the curving space of control itself.

Let us take one example that particularly addresses the computation of topological relations as a generalized instance of postcybernetic control. Parametric design,[11] for example, can be said to underpin many forms of topological operationality as it specifically works to programme mathematical relations between data sets. As the term ‘parametric’ implies, a parameter is a variable to which other variables are related. Hence sets of variables and their relationships determine the changes of a spatial form. While the initial conditions of the parametric design are still programmed through a binary logic of 0s and 1s, these conditions are open to change through the evolutionary processing of parameters, when new variables are at once generated from and added to the set of initial values. Hence, the continual relation of programmed variables is more important to the parametric design of urban space, for instance, than the digitalization of physical variables into sets of 0s and 1s. This means that while, on the one hand, parametric relations order variables into sequential binary sets, they are also determined by the qualitative level of topological functions, where differential relations explain how the transformation of one value is equivalent to the continual variations of the whole space.

Nevertheless, the determination of a continual correspondence between data variables and the form of space is not specific to parametric design. As Sanford Kwinter (2008: 37) points out, design has always been a highly advanced form of rationality. Design is a rational technique, which breeds and mutates infrastructures: from knowledge to cultural and urban infrastructures. Thus parametric design is just another instance of design as a logistics of operations, where algorithmic information and data structures are now ‘oriented to performative environments, to protocols, and, *in extremis*, to psychological operations’ (2008: 39). According to Kwinter, as architecture has turned into ‘experiments in design logic, research and potential’ (2008: 51), so has the computational paradigm extended concepts of materiality, society, economics and nature into the incorporeal field of intensive manifolds, turning spaces into ‘shapes of time’ (2008: 53). As the qualitative level of relations (or topological continuity) has become central to computational design, so time, intended as lapses of evolution, growths, adaptation of initial values, has come to determine the final shape of spaces.[12]

This has also meant that with parametric design, modifications of values can be performed almost in real time, compared to the time-consuming re-drawing required by the traditional AutoCAD for instance. Before the advent of parametric design, buildings were modelled using computer drafting programs, such as industry standards AutoCAD or MicroStation, and then analysed by engineers using their own software, and ultimately sent to environmental engineers using yet another software program. Parametric design affords the engineering of the overall levels of a spatial form to be manipulated at the same time. Through the altering of specific parameters that are able to automatically adjust by building on data such as the total gross area, total building height, total number of floors, the various levels of engineering are integrated into one topological software program. Parametric design offers the modulation of variable relationships between entities, where the alteration of properties results in different outcomes of the overall form.[13] Parameters can be established from a vast list of possibilities; they could be taken from data on wind speed or rainfall for example. These variabilities can also be directly related to costs on a spreadsheet ultimately ensuring a smooth direct relationality between architectural and economic changes. This direct relation between financial costs and spatial form partakes of a topological regime of immediate convergence – or algebraic invariant – between variables of forms and economic value.[14] To establish continuity between discontinuous groups of values, one part of the design has to respond to transformations in another, or the entire design can respond to changing conditions, such as light, airflow, but also weight distribution and gravitational pressure. In general, any output or variable from the outside is pre-included in the list of possibilities of the algorithmic architecture, defining space as a topological engine of potentialities. Results can be instantaneously fed back into the system through a recursive loop of algorithms, tested and played again to evolve different results.

As Michel Hensel and Achim Menges (2009: 212) argue, parametric architecture needs to be conceived as a system with a set of finite internal relationships and external forces that inform it and to which it responds. These relationships are constructed by the computational capacities to envisage the material characteristics and behaviour of locally specific and yet dynamic environmental conditions, which for instance produce microclimatic levels of differentiation in a geographic field. In general, the shift from computational programming as ‘design-defining’ (e.g. design based on pre-set algorithms) to design as ‘program-evolving’ (e.g. design derived from the interaction between parameters that become generative of other levels by responding to real-time inputs) explains how design now relies on continual relations rather than digital fixing.[15] It is precisely this emphasis on the evolving relation between parameters of interaction that now characterizes computational architecture in terms of real-time adaptation, emergence and change. From this standpoint, program-evolving urbanism includes the design of smart infrastructures that are able to monitor, respond to and/or anticipate the transport logistics of a city (including roads, rail, water and air) for instance. As parameters have become evolutionary variables that enter and exit relations with other parameters, urban design is set to include time-related data in the programming.

The integration of wireless sensor networks into large-scale engineering systems, such as, for instance, networks of pipelines, tunnels and bridges, relies on the parametric programming of engineering systems that directly respond to sensor networks.[16] The generative programming of parameters, whereby each parameter includes temporal variations, now animates the design of urban infrastructures integrating differential relations between systems (rail, road, air, water systems) into one smooth machine of continual variation. Here the monitoring of real-time data, central to software-enhanced infrastructure, is only another facet of a program-evolving urbanism where smooth, speedy and cost-efficient systems are integrated into an evolving meta-system including all infrastructures.

The scope of program-evolving urbanism is not dissimilar to computer devices offering us new possibilities of navigation, which then become part of our saved favourite paths, presenting us with set solutions, which we have previously selected or added to the navigation program. In other words, in the same way as your smart phone works as a monitor device for tracking your location, which then becomes data used to construct the profile of your movement, so the monitoring procedure of smart infrastructures collects data which then become part of the programming of new infrastructural systems. As data become recorded so they evolve into predictive scenarios aiming not simply at pre-setting your movement but generating its future conditions through the evolving interaction of parameters with real-time data. Ultimately, the goal of parametric design is *deep* relationality, the *real-time* integration of the evolving variables of a built environment in software systems able to create scenarios by responding to or pre-adapting data. As Neil Leach (2009) has observed, parametric architecture needs to be conceived as marking a third phase in digital design, where the use of algorithms to experiment with forms and the tectonic application of digital software are being superseded by the software evolution of urban space.[17] This means that software is no longer a tool for design, but it has become one *with* design and its form of rationality, now seemingly operating through the generative parameters of continual variation.

In sum, parametric design is an instance of the general tendency towards topological control, where changes, or the evolution of parameters, are already pre-programmed through the invariant function between parameters. The aesthetic appeal of smooth control is precisely the continuous curvature of the straight line, the rounded shapes of a folding urbanscape of temporal variations and real-time responsiveness.

From this standpoint, change is intrinsic to the operative logic of control to the extent that it is pro-actively programmed or actively programmed within the codes that guarantee continuity of form and function. The invariant connection between the distinct levels of networks is instantiated in parametric urban models, which are based not on geometric planning but on the mathematical variables of evolutive urban software. As *R&Sie(n)* architect François Roche (2009: 40) recently suggested, the new parametric programming of digital cities resembles less a binary grid of finite sets (0s and 1s) than a biostructure that develops its own adaptive behaviour, based on growth scripts and open algorithms. This is a new bio-computational design whose programming capacities are stretched to calculate potential conditions of relationality and intensive change, rather than writing scripts of what can eventually emerge. These design programs are meta-protocols constituting an urban ecology of software continuities between many discrete infrastructural systems integrated into a single envelope – an intensive manifold whose interior and exterior sites can be activated in any number of ways. From this standpoint, parametric design may become an example of how urban infrastructures are co-evolving with urban software such that the invisible architecture of topological computing is no longer set to represent but to programme the development of physical space.

Temporal Qualities

The introduction of temporal qualities into parametric design characterizes the aesthetics of curvature. Here relations between parametric quantities shape parts into an architecture of the whole.[18] The topological approach has substituted the function of digital sequencing with the composite function of relations, so that changes at one level of parametric value effectuate changes at another level. From this standpoint, parametric design has given way to a plethora of morphogenetic

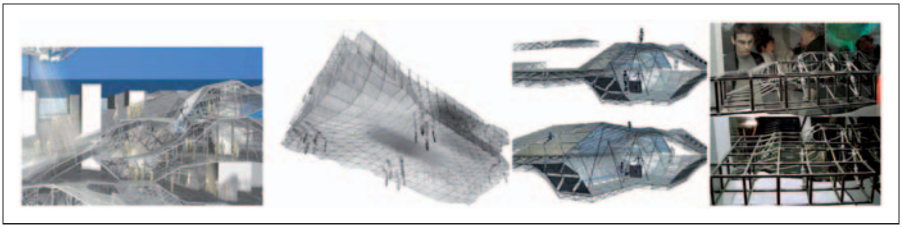

Figure 1. *Topotransegrity*, 2006.
Source: 5Subzero (Delphine Ammann, Karim Muallam, Robert R. Neumayr, Georgina Robledo).

architectures, where the whole stems from the relations between mechanical, physical and algorithmic parts.

For instance, *Topotransegrity*, an award-winning responsive and kinetic architecture designed by 5Subzero,[19] shows how the spatial organization of public space can derive from a topological design of continual adaptation between software programs, mechanical parts and real-time physical movement. *Topotransegrity* brings together surfaces through pneumatic space frame structures that can be manipulated either through an automated control mechanism, through real-time feedback or by software programs. In particular, the programming of the structure relies on external and internal parameters defined by the environment influencing different parts of the structure. The continual relation between the parameters and their changing mode of operation affords a series of emerging user-dependent spatial configurations.

As a whole, *Topotransegrity* is a kinetic structure, sustained by three sets of pneumatic pistons designed by Festo. The pistons are equipped with responsive software, which evaluates the surroundings and reconfigures the structure according to changing conditions. *Topotransegrity* extends across existing buildings at the Barbican complex in London to form a topological surface of connection. This surface constitutes a generic responsive structural system ready to adapt to distinct spatial requirements. The structure is capable of various transformations, which range from small-scale surface articulations to large surface deformations, working as temporary enclosures. The introduction of contingent elements from the environment into the parametric programming of its different parts is here used by the 5Subzero group as a trigger that allows the responsive structure to multiply, intensify and vary the potential uses of public spaces. According to the 5Subzero group, *Topotransegrity* is therefore not simply a pre-programmed structure, but has to rely on external real-time feedback to generate new internal configurations. For instance, sensors, input devices and wireless networks are integrated into existing building materials transforming the architectural space of the Barbican into a complex continuity. This is determined by invariant functions deploying the topological relation between the program mode (parameters automating the basic functions of the structure by adding new levels of connection), the crowd mode (parameters determined by real-time responses of the structure towards movements and behavioural patterns of visitors) and the memory mode (parameters that record on a long-term basis the paths and motion patterns chosen by users). These three parametric modes of operation run simultaneously, interacting with visitors in a permanent feedback loop: local reactions to spatial adaptations are fed back into the system of parameters, which in turn specifically re-designs the built environment according to changing patterns of use.

It could be argued that the crowd or any other external data constitute those contingencies that are somehow controlled or directed by the program, which then spatializes the qualities of temporal variations. It may be true then to say that *Topotransegrity* is unable to create the conditions for a radically novel reconfiguration of space to the extent that contingencies serve the software system merely to find optimal solutions to emerging problems. On the other hand, however, *Topotransegrity* is precisely an instance of a topological aesthetics of control turning discrete points and finite lines into a mesh of infinitesimal points of variations governed by invariant functions, which integrate distinct actual parameters into a continual surface of configurations. This is less about the software hierarchical mastery over hardware or modes of behaviour than a form of control defined by the differential integration of the temporal qualities of software programming, kinetic mechanics and real-time interaction. *Topotransegrity* therefore points to topological aesthetics as a dominant form of spatial experimentation in postcybernetic culture. But can this aesthetics be traversed by cut-bringing events irreducible to the pre-emptive program of parametric control?

Eternal Quantities

Alfred North Whitehead's notion of mereotopology (1978: 294–301)[20] proposed that space is composed of actual entities that connect.[21] These are atomic occasions or discrete events explaining how the becoming of continuity and not continual change occurs. Zeno's paradox of discrete units and infinitesimal divisibility is here not addressed through the Bergsonian metaphysics of a continual duration, or *élan vital*, where all quantity amounts to a difference in kind.[22] A mereotopology of atomic spatio-temporality instead explains that potentials break the continuity of connection. According to Whitehead, Leibniz's infinitesimal divisions, which Poincaré defined as topological invariants, could not explain the reality of events on the plane of continuity (or the continual chain of cause and effect determining the sequential relations between actualities), because the distance between actualities could not be filled by the infinitesimal continuity of percepts and affects (Whitehead, 1978: 332–3). On the contrary, the distance between actual entities had to be considered as such: a space of connection, overlapping, inclusion, juxtaposition, disjunction and intersection defined by the points and lines of finite actualities. In other words, there are always actualities amid actualities.

According to Whitehead (1978: 328), the relations between actual occasions need to be compared not to the infinite lines of the Euclidean parallel axiom, but to finite segments. Each actual occasion is finite. It does not change and does not move. Actual entities, like the parameters in *Topotransegrity*, are real potentialities, determined by what Whitehead (1978:169) calls causal efficacy, the sequential order of data defined by the physical prehensions of past data from one entity to the next. From this standpoint, the continuity between parameters is explained by the connection between entities, which are not geometrical points but 'spatial regions' with semi-boundaries (e.g. volumes, lumps, spheres) (1978: 63, 121–5, 206). Hence, continuity is not explained by infinitesimals or the convergence of two parallel infinite lines touching infinity, but by the relation between these spatio-temporal regions of objectified real potentialities (actual entities) that are slices of time, atomic durations (1978: 77).[23] Instead of infinitesimally divisible points of perceptions and affections, Whitehead believes that there is an infinite number of actual entities between any two actualities, even between those that are nominally close together. This is why Whitehead rejects Zeno's paradox of infinitesimal small points and argues that continuity is not a ground to start from, but something that has to be achieved as a result of actual entities' extensive connections (1978: 96–7, 294).

From this standpoint, the mereotopological relation between distinct sets of parameters deployed in *Topotransegrity* corresponds to the real potential of each actual entity to become the datum of another parameter. In other words, since the topological relation between parameters implies that a change in a parameter has an effect on other parameters and a generalized impact on the whole architectural structure, each parameter can be considered to have a real potential to become data for change for another. On one level, the extensive subdivisions (the parametric connection between software, crowd and memory modes) and the topological relations of the points and lines between the physical space, the digital software and the kinetic pistons compose the *real potential* of actual entities (finite quantities or parameters). This actual level of parametric quantification and relationality describes the real potential of extended continuity, where the relation between finite entities is intersected by other finite entities and not by the phenomenal qualities of perception and affection. The parametric design of *Topotransegrity*'s project therefore deploys a nexus of actual entities or events, which, according to Whitehead, stems from a series of sequences constituting a 'historic fact' (the objectified real potentials of software, crowd and memory parameters at each spatio-temporal connection) relating occasions to occasions (1978: 66). Data are what has been in the past, but also what might have been, and what might be of the spatial configurations: a software program, the real-time movements of a crowd, the reshaping of the pistons. All these data are always actuals, and their specific potentiality is always a real possibility that affects the next series. Following the logic of cause and effect, the relation between parametric data involves a movement from past to present and future spatio-temporalities.

The parametric software of the adaptive structure, determined by constant feedback loops with the movement of the crowd and the kinetic configuration of the pneumatic pistons, operates in the same repetitive fashion of physical, organic and inorganic, matter. Here the invariant function of the topological continuum corresponds to the physical, extensive connection between actual entities, the overlapping and intersection between parts (as defined by mereology). This is only the topological level of parametric design. But a mereotopological reading of *Topotransegrity* will have to include another level of relationality, an abstract set of infinite relations, which cannot be defined exclusively in terms of physical qualities. *Topotransegrity* in fact operates on two levels of potentialities, which may correspond with Whitehead's distinction between the real potential of each actual entity to become the datum of another and the pure potentials (or eternal objects) which ingress actual occasions in many points (1978: 23). But the level of pure potentials in *Topotransegrity* is not explicitly unfolded although it constitutes the modes of partition, separation or quantification of qualities of movement and response. From the standpoint of mereotopology these modes imply at least two orders of magnitude, the order of actual quantities and the order of infinite quantities.

A mereotopological view of *Topotransegrity* indeed can be taken as an example of parametric control, precisely involving the constraints of these two orders of relational quantities, not only corresponding to the continual programming of contingencies but to the discontinuity between control and events. For mereotopology is a symmetry-breaking schema of real and pure potentials that explain how the continuous connection between actualities is infected with abstract objects, whose indeterminate reality adds new character to actual relations. This is not an eternal geometry operating on contingent physics. Despite the fact that the order of eternal objects, as pure relata, is not open to be modified by eventful actualities, the objects themselves become nonetheless part and parcel of events. In particular, it is the way that these otherwise non-communicating objects are selected that allows them to acquire unity in actual entities. This new unity reveals how eternal objects undergo eventful changes and are indeed intrinsic to actualities. This also means that events are at once disjunctions of actual data and conjunction of eternal objects.

The topological model implies a continual ground by which events are such only when it becomes possible for actualities to jump out of the spatio-temporal grid into the infinity of virtual time. The mereotopological schema instead suggests that events are the cumulative order of spatio-temporal actualities hosting an unrepeatable togetherness of eternal objects. Therefore it is not the formal hierarchy of eternal objects that determines actual events. Events instead are the result of the actual accumulation of physical data, whose causal chain is interrupted by the ingression of eternal objects. These are not simply selected by actualities to manage orders of behaviour or action, but are prehended for the pure chance or potentialities that these objects offer. Actualities therefore do not simply operate a probabilistic calculation about which eternal object to select. On the contrary, selection is a *feeling* for non-actual ideas, involving the ingression of chance for what has happened, what may happen and what could have happened. This is how contingency becomes intrinsic to the speculative power of eternal objects: a process by which existing relations can change character and become anew. This means that the indeterminacy of eternal objects is felt like the reality of chance, pure potentialities, determining the atomic (and eventful) character of actual relations.[24]

There is no undifferentiated pool of eternal objects constituting a continuum of temporal qualities, divided or spatialized by actual entities. On the contrary, each eternal object uniquely contributes or adds indetermination to each set of actual entities in so far as each eternal object 'stands a determinateness as to the relationship of *A* [an eternal object] to other eternal objects' (1978: 160). Eternal objects explain internal relations as 'a systematic mutual relatedness' where each eternal object has a status (1978: 161). Eternal objects are not temporal forms of relations but are permanent and infinite sets of eternal objects, isolated from their individual essences. They are relata in the uniform schema of relational essences, where each eternal objects stands internally within all of its possible relationships (1978: 164). Whitehead explains that there is a uniform scheme of relationships between the infinite sets of eternal objects, which acquire a togetherness of their individual essence once they are included in an actual entity. This means that for any actual occasion 'a' there is a group of eternal objects ingredient in that actual occasion. Since any given group of eternal objects may form the base of an abstractive hierarchy of relation, there is an abstractive hierarchy associated with any actual occasion 'a'. This associated hierarchy is 'the shape, or pattern, or form, of the occasion, insofar as the occasion is constituted of what enters into full realization' (1978: 170).

Each actual parameter is then infected by a multiplicity of eternal objects and becomes related to other parameters by means of their potentiality to be selected by an infinite number of actualities. And yet, eternal objects do not add intensive temporalities to parameters. On the contrary, actual parameters are the point of selective limitation or constraint of these infinite objects, and as such they are general determinations applied to the spatio-temporal continuum. 'Thus primarily, the spatio-temporal continuum is a locus of relational possibility, selected from the more general realm of systematic [and abstract] relationship' (1978: 161). Once eternal objects are selected they add a new level of determination to the spatio-temporal sequence of parameters, a novel character to the actual relations between quantities of systematic length, weighted with the individual peculiarities of the relevant environment. For Whitehead, the mereotopological schema explains how novelty involves a discontinuity of continual relations. Any parameter, insofar as it is an actual entity, corresponds to the prehensions of physical data of past, present and future actualities. But a parametric value is also a conceptual prehension of the abstract relata or eternal objects, which are included in the actual parameter as gradients of determination.

If Bergson's *élan vital* is a virtual continuum each time divided by perceptual selections or material actualities, Whitehead seems to claim that this correlation between one time (the topological invariant continuum of indiscernible, undifferentiated duration) to many spaces precludes any event ever occurring on the extensive continuum of actualities. Like Henri Poincaré's view of an infinitesimal curving space or a topological continuum of uncut forms, Bergson (1991: 133–78) was seeking a temporal invariant between events. From this standpoint, only *virtual* time (uncoordinated intensive time) can *amodally* link two causally connected actualities (or parameters). Such virtual time is a real interval, exposing the plenitude of cosmic time, which has no intrinsic measure except a continual variation of differential relations. Instead, Whitehead's mereotopological schema defines the relationship between actual entities as marked by the cut that the abstract infinity between eternal objects adds to the physical chain.

A parameter is not only the transduction of physical qualities (such as the volume of a space, gravitational forces, the circulation of air, the movement of people, the shades of lights, the sonic frequencies, the electromagnetic vibrations, etc.) into finite quantities, but an actual object itself. There is, however, an abstract potential within parameters that cannot be grasped at the level of sequential sets but needs to be explained as the infinite quantities of eternal objects that infect and add novelty to actual parameters. This means that the invariant function of topological continual relations, grounding the ontological dominance of the aesthetics of curvilinearity, is only one way of articulating the relation between control and events. The mereotopological schema of eternal objects and actual entities offers another way.

The dominance of the invariant function determines relations between parameters in terms of vectorial qualities. As demonstrated by Greg Lynn's calculus-based architectural forms, it is the qualitative relations of vectors that constitute space as a fluid environment of forces. But this qualitative inflection of parametric design has become a dominant post-cybernetic procedure of connecting entities through a temporal flux of continual variations. For instance, the aesthetic appeal of morphogenetic forms defined by the continual variation of points into temporal vectors, has become equivalent to the aesthetic power of control transmuting actualities into supple lines of convergence, compatibility and uniformality.

One cannot deny that parametric design includes non-exactly measurable qualities into programming, thus conferring a qualitative transformation of the geometrical form as a whole, specifically resulting from the operations of a differential relation encompassing all points on a curve. What is suggested here, however, is that the qualitative dimension of the differential relation has become central to the topological view of the postcybernetic logic of control, whereby prediction is no longer based on the calculation of finite probabilities, but on the inclusion of potential qualities. Brian Massumi (2007) has defined this shift in terms of the mediatic power of pre-emption, whereby the indeterminate qualities of the future are incessantly foreclosed into sets of probabilities in the present. The ingression of topological invariants into cybernetic systems precisely allows automated processes to constantly transduce temporal qualities into quantities, by developing an aesthetic of continual variability of quantities.

And yet, one cannot overlook the process of quantifications of which parametric design is an instance. To argue that this mainly entails a transduction of qualities into quantities in a fluctuating geometrical shape is to deny that quantities could ever be more than finite sets of instructions. Whitehead's mereotopological schema instead adds an abstract schema of discontinuous objects to the actual continuum, so that infinite relations between pure quantities can ingress actual qualities. Points of connection are not only finite parts that overlap, the process of overlapping also includes the selection of abstract quantities that add a new quantitative character to overlapped parts. To put it in another way, parametric relations are not only transductions of qualities into quantities. They are infected with abstract non-denumerable relations of pure quantities, eternal objects: discrete yet permanent relations adding novel character to existing parametric relations. From a mereotopological point of view, each parametric extensive relation is hosting another order of quantities that cannot be contained by the number of its actual members.

If the topology of parametric design implies the calculation of variables through the invariant function, Whitehead's mereotopology always exposes actual events escaping any form of overall continuity. Mereotopology then suggests that underneath continual morphogenesis, there lies a holey space of abstract quantities, infinite relations of numbers that cannot be counted as such. These are the black holes of probability and statistical calculation, remarking the occurrence of *something* travelling beneath and throughout actual regions. These holes in parametric design define the intrusion of parasitic quantities, non-isomorphic functions unable to unite all finite quantities into a morphogenetic continuity.

The topological view of the digital processing of physical data has already unleashed these abstract quantities into culture through the parametric design of buildings, cities, environments, animate and inanimate objects. This design indeed involves not simply the algebraic manipulation of physical data, but the computation of the extensive continuum of actualities (physical and digital parameters) involving their irreversible encounter with abstract quantities, adding uncomputable chance to actual relations. Parametric design is then also an instance of an aesthetic of discontinuity between abstract objects and between actual sets. This discontinuity explains how the spatio-temporal continuum can become other than the actual relations composing it. Here, the introduction of novel configurations of space is not derived from the continual variations of form, but from a universe of discontinuous potentialities abducting the actual relations of data and thus exposing parametric aesthetics to the infinite quantities accompanying any set of probabilities. If topological continuity is the aesthetic design of postcybernetic control via the continual variation of qualities, mereotopological discontinuities expose the aesthetics of irreducible quantities defining the event of computational relations beyond the smooth surface of pre-emption. Since parametric design deals with different orders of quantification (finite and infinite relations), it cannot but become a channel for the proliferation of uncomputable realities within the programming of extensive relations. The parametric aesthetics of *Topotransegrity* therefore does not simply offer a formal system of relations between the software level of programming, the hardware level of the kinetic pistons, the level of physical movement, the level of circulation of air and access. On the contrary, mereotopology exposes this formal system to indeterminate, uncomputable and contingent potentialities of urban programming, where indeterminate quantities invade existing parametric relations. It is this abstract quantitative order of relations that needs to be accounted for in debates about the significance of topology for the aesthetics of digital design.

Notes

1. Henri Poincaré is considered to be the originator of algebraic topology and of the theory of analytic function. In 1895, he published *Analysis Situ*, one of the earliest systematic theorizations of topology. In particular, Poincaré's use of 'homotopy theory' contributed to reducing topological questions to algebra by associating topological spaces with various groups defined as algebraic invariants. Poincaré introduced a fundamental group to distinguish different

categories of two-dimensional surfaces. He was able to show that any two-dimensional surface, having the same fundamental group as the two-dimensional surface of a sphere, is topologically equivalent to a sphere. He conjectured that the result held for three-dimensional manifolds and could be extended to higher dimensions. Yet up to the present there still is no list of possible manifolds that can be checked to verify that they all have different homotopy groups. The invariant function, as a property of non-change, explains change as the morphological transformation of the whole, rather than as parts breaking from the whole. See Boyer (1989: 599–605).

2. Patrick Schumacher recently claimed that parametricism is the dominant style of today's avant-garde and insists on the power of large-scale urban schemes. See Schumacher (2009).
3. In the mathematical field of topology, a homeomorphism or topological isomorphism or bicontinuous function (from the Greek words *Ὅμοιος* [*homoios*] = similar and *μορφή* [*morphē*] = shape, form) is a continuous function between two topological spaces that has a continuous inverse function. Homeomorphisms are the isomorphisms in the category of topological spaces (e.g. the mappings which preserve all the topological properties of a given space). Two spaces with a homeomorphism between them are called homeomorphic. From a topological viewpoint they are the same. If topological space is a geometric object, for instance, homeomorphism defines a continuous stretching and bending of the object into a new shape. Thus, a square and a circle are homeomorphic to each other, but a sphere and a donut are not. In other words, topology is the study of those properties of objects that do not change when homeomorphisms are applied. As Henri Poincaré famously said, mathematics is not the study of objects, but instead the relations (isomorphisms for instance) between them. See Boyer (1989: 603–4).
4. Calculus stems from the manipulation of very small quantities or infinitesimal objects that can be treated like numbers but which are 'infinitely small'. On a number line, infinitesimals have not location zero, but have zero distance from zero. Such quantity corresponded to a single number. As Boyer (1989: 216) explains, only after the development of a general abstract concept of real number was it possible to interpret the differential calculus in terms of the limit of an infinite sequence of ratios or numbers.
5. Infinitesimals have been used to express the idea of objects so small that they cannot be seen or measured. An infinitesimal number is a non-standard number whose modulus is less than any non-zero positive standard number. In mathematics, an infinitesimal, or infinitely small number, is a number that is greater in absolute value than zero yet smaller than any positive real number. An infinitesimal is a variable whose limit is zero. The development by Abraham Robinson (1960) of 'Nonstandard Analysis' conferred new significance on infinitesimals and brought them closer to the vision of Leibniz (1646–1716), who introduced the dy/dx notation for the derivative and perceived infinitesimals more like small but constant quantities. Infinitesimal or differential calculus is an area of mathematics pioneered by Gottfried Leibniz based on the concept of infinitesimals, as opposed to the calculus of Isaac Newton, which is based upon the concept of the limit. See Boyer (1989: 391–5, 519–22).

6. Monads are 'substantial forms of being'. They are eternal, indecomposable, individual, subject to their own laws, un-interacting, and each reflecting the entire universe in a pre-established order. Monads are centres of force, while space, matter and motion are phenomenal. In 1960, Abraham Robinson worked out a rigorous foundation for Leibniz's infinitesimals, using model theory. With non-standard analysis, Leibniz's mathematical reasoning was also revised. See Martin and Brown (1988).
7. The law of continuity is based on the principle that between one state and another there are infinite intermediate states. A continuous entity – a *continuum* – has no interior 'gaps'. On the contrary, to be discrete is to be separated, like the scattered pebbles on a beach or the leaves on a tree. Continuity connotes *undivided* unity; discreteness, divided plurality. Repeated or successive division gives the fundamental nature of a continuum. The process of dividing a continuous line into parts will never terminate in an *indivisible part or atom* that cannot be further divided. One of the first formulations of the law of continuity is the famous Zeno's paradox, a set of problems devised by Zeno of Elea. To support Parmenides' metaphysical doctrine, that 'all is one' contrary to what we perceive, Zeno's paradoxes demonstrate that plurality and change are illusions. Parmenides rejected pluralism and the reality of any kind of change: all was one indivisible, unchanging reality. Another formulation of the law of continuity is offered by Leibniz (see his preface to *New Essays on Human Understanding* (1981 [c. 1704]). The law of continuity in Leibniz also refers to the principle of pre-established harmony, according to which each event occurs when it does because it was pre-programmed to do so by God. See Boyer (1989: 74, 399–407).
8. However, Leibniz is thought to have resolved the paradoxes of continuity by arguing that there are no jumps in nature and thus no discontinuities. He believed that any change passes through some intermediate change and that there is an actual infinity in things. Similarly, he used this principle of continuity to show that no motion can arise from a state of complete rest. See Leibniz (1981, 2001).
9. A derivative is the quotient of two differentials, a differential relation such as dy/dx, where dy and dx are infinitely small quantities whose relation to x (or the quantity of the ordinate) and y (or the quantity of the abscissa) is equal to zero. But, whereas the relation between the actualities x and y is equal to zero, the relation between the two infinitely small quantities (dx and dy) is not zero. This means that these infinitely small quantities are of another existing order compared to the actualities x and y. These infinitely small quantities persist as they vanish by approaching zero ($dx/dy = 0$).
10. Gilles Deleuze's reading of Leibniz's infinitesimal calculus explained that the relation between x and y could not but correspond to another kind of relation describing the differential distance between dx and dy. While dx and dy cancel each other out in the form of vanishing quantities (infinitesimals), the differential relation between them remains itself real. From this standpoint, both Leibniz and Deleuze link the mathematical problem of infinity to the geometrical problem of deriving the function of a curve (the relation between x and y quantities) from the given property of its tangent. See Deleuze (2004: 217–20).

11. ‘Parametric’ is a term used in a variety of disciplines from mathematics through to design. Literally it means working within parameters of a defined range. Within the field of contemporary design, it refers broadly to the utilization of parametric modelling software. In contrast to standard software packages based on datum geometric objects, parametric software links dimensions and parameters to geometry, thereby allowing for the incremental adjustment of a part, producing effects on the whole assembly. For example, as a point within a curve is repositioned the whole curve comes to realign itself. Parametric software therefore lends itself to curvilinear design as in the work of Frank Gehry, Zaha Hadid and other formal architects. However, it would be wrong to assume that parametric design is concerned primarily with form-making. On the contrary, parametric techniques afford design new modes of efficiency compared to standard approaches, and new ways of coordinating the construction process (e.g. Business Information Modelling), as in the case of Digital Project, an architectural version of CATIA customized for the building industry by Gehry Technologies. See Meredith (2008).
12. The correspondence between qualitative change, temporality and movement is evident in the use of computed animation in the design of spatial fields of relations as well as in the design of real-time interactive architectures, where environmental factors and users can become inputs that change the programmed structure of parameters and algorithms. On the notion of time-like architectures see Lynn (1999: 9–41); see also Grosz (2001), Spuybroek (2004) and Senagala (2001).
13. For example, a line has two parameters – its length and its direction – and altering one of these factors gives you a different form. A polyline has the previous two factors plus the positioning of its vertices and, if any of these is altered, a different form is given, and so on.
14. For example a tower that has a vertical rotation of floor plates can be seen in terms of cost: a very twisted form costs more than a not so twisted form.
15. Among some of the most recent experiments with designing program-evolving architectures, the work of artist Casey-Reas on software processing particularly engages with the evolving capacities of variables exploring the microdynamics of emergent form out of complex levels of urban interaction. See ‘Intensive Fields – New Parametric Techniques In Urbanism’, USC Conference, Los Angeles, 12 December 2009. See http://parasite.usc.edu/?page_id=28 (accessed May 2012).
16. The Infrasense Laboratory at Imperial College, London, has recently started a research project called ‘Smart Infrastructure: Wireless Sensor Networks for Condition Assessment and Monitoring of Civil Engineering Infrastructure’. This form of smart infrastructure wireless sensor networks is here used above all to monitor changes and collect data that software can analyse so as to look for new solutions to emerging problems, such as the flow of water due to leaking pipes, for instance. See: http://www2.imperial.ac.uk/infrasense/SmartInfrastructure.php (accessed April 2011).
17. Leach defines the first instance of digital architecture as the phase of virtual reality, defined by early experimentation of digital forms. In 2002–3, a

second phase of digital design opposed to the earlier phase of form-making produced an emphasis on the notion of tectonics because the materials of architecture had become increasingly informed by the worlds of the computer. In particular, he refers to the computational programming of the British Museum roof. A third shift in digital design is marked by the current use of computation at an urban scale, defined by the development of parametric techniques in the design of cities. See Leach (2009).

18. One can take as an example of the aesthetics of the curvature Zaha Hadid Architects' design of the BMW Central Building, where the primary organizing strategy of the building lies in the scissor-section that connects ground floor and first floor into a continuous field: two sequences of terraced plates (like giant staircases) step up from north to south and from south to north. See: http://zahahadidblog.com/projects/2007/06/11/bmw-central-building (accessed October 2010).
19. See: www.5subzero.at (accessed October 2010).
20. The analysis of parthood relations (*mereology*, from the Greek *mero*, 'part') was an ontological alternative to set theory, which dispensed with abstract entities and treated all objects of quantification as individuals. As a formal theory, mereology is an attempt to set out the general principles underlying the relationships between a whole and its constituent parts, just like set theory is an attempt to set out the principles underlying the relationships between a class and its constituent members. Mereological reasoning, however, cannot by itself explain the notion of a whole (a self-connected whole, such as a stone or a whistle, as opposed to a scattered entity of disconnected parts, such as a broken glass, an archipelago, or the sum of two distinct cats). Whitehead's early attempts to characterize his ontology of events provide a good exemplification of this mereological dilemma. For Whitehead, a necessary condition for two events to have a sum is that they be at least 'joined' to each other, that is, connected (be they discrete or not). These relations, concerned with spatio-temporal entities, cannot, however, be defined directly in terms of plain mereological primitives. To overcome the bounds of mereology, the microscopic discontinuity of matter had to be overcome since the question of what characterizes objects that are all of a piece required topological analysis. Two distinct events can be perfectly spatio-temporally co-located, they do not *occupy* the spatio-temporal region at which they are *located*, and can therefore share it with other things. The combination of mereology and topology contributed to Whitehead's articulation of the notion of the extended continuum. See Whitehead (1978: 294–301).
21. Whitehead used the notion of mereotopology to address the problem of abstraction and spatial measurement without equating abstraction to infinitesimal points. He used the logic of non-metrical spatial relations of extensive parts and wholes, thus starting with concrete actualities or occasions of experience. Since all metrical relations involve measurement and to measure or quantify is the ultimate method of abstraction, Whitehead developed the notion of extensive abstraction. This notion was intended to problematize the general theory of relativity and the theory of measurement, which seemingly collapsed physics and geometry, ignoring, according to Whitehead, the distinction between the abstract and the concrete. For Whitehead it was

instead necessary to separate the geometrico-mathematical order for the physical world so as to formally be able to explain their relations, thus making measurement as determinate as possible. According to Whitehead, the general theory of relativity equates the relational structures of geometry with contingent relations of facts and thus loses sight of the logical relations that would make cosmological measurement possible. This is why Whitehead's mereotopological approach insists on the spatialization of extension and the temporalization of extension, whereby 'physical time is the reflection of genetic divisibility into coordinate divisibility' (1978: 289). Whitehead argued that the solution to this problem was to separate the necessary relations of geometry from the contingent relations of physics, so that one's theory of space and gravity is 'bimetric', or is built from the two metrics of geometry and physics. See Whitehead (1978: 283–7, 294–301, 327–9).

22. In particular, and contrary to Whitehead, Bergson's theory of time, the qualitative time of the *élan vital*, is opposed to the metric time of scientific epistemology, thus identifying the necessity of abstraction with the imperatives of the scientific enterprise. Whitehead, on the contrary, seeks a divergence between geometrico-mathematical abstraction and physical actualities in order to propose a more rigorous metaphysical schema of relations. See Bergson (1994: 358–65, 374–80).
23. As Whitehead explains, each actual entity is atomic as it is spatio-temporally extended (1978: 77).
24. As Whitehead specifies: 'In the essence of each eternal object there stands an indeterminateness which expresses its indifferent patience for any mode of ingression into any actual occasion' (1997: 171).

References

Bergson, Henri (1991) *Matter and Memory*. New York: Zone Books.

Bergson, Henri (1994) *Creative Evolution*. New York: Random House, the Modern Library.

Boyer, Carl B. (1989) *A History of Mathematics*, foreword by Isaac Asimov, 2nd edn. New York: John Wiley and Sons.

Bratton, Benjamin H. (2009) 'iPhone City', in Neil Leach (ed.) *Digital Cities*, special issue, *Architectural Design* 79(4): 90–7.

Chaitin, Gregory J. (2001) *Exploring Randomness*. London: Springer-Verlag.

Deleuze, Gilles (2004) *Difference and Repetition*, trans. Paul Patton. New York: Continuum Press.

Fletcher, Peter (1989) 'Non-standard Set Theory', *Journal of Symbolic Logic* 54(3): 1000–8. URL (consulted October 2010): http://www.jstor.org/stable/2274759.

Grosz, Elisabeth (2001) *Architecture from the Outside*. Cambridge, MA: MIT Press.

Hensel, Michel and Achim Menges (2009) 'The Heterogeneous Space of Morpho-ecologies', in Michel Hensel, Christopher Hight and Achim Menges (eds) *Space Reader: Heterogeneous Space in Architecture*. New York: John Wiley and Sons.

Kipnis, Jeffrey (2009) 'Towards a New Architecture', in Michel Hensel, Christopher Hight and Achim Menges (eds) *Space Reader: Heterogeneous Space in Architecture*. New York: John Wiley and Sons.
Kwinter, Sanford (2008) *Far from Equilibrium: Essays on Technology and Digital Design*. New York: Actar Press.
Leach, Neil (2009) 'Introduction to Intensive Fields: New Parametric Techniques in Urbanism', paper presented at the USC Conference, Los Angeles, 12 December. URL (consulted August 2010): http://arch-pubs.usc.edu/parasite/intensive-fields/video-archive/
Leibniz, W. Gottfried (1981) *New Essays on Human Understanding*, trans. Peter Remnant and Jonathan Bennett. Cambridge: Cambridge University Press.
Leibniz, W. Gottfried (2001) *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672–1686*, trans., ed. and with an introduction by Richard T.W. Arthur. New Haven, CT: Yale University Press.
Lynn, Greg (1993) 'Architectural Curvilinearity. The Folded, the Pliant and the Supple', *Architectural Design* 63(3/4): 8–15. URL (consulted June 2010): http://bscw.archlab.tuwein.ac.at/pub.cgi/d616115/Lynn_e.pdf.
Lynn, Greg (1999) *Animate Form*. Princeton, NJ: Princeton University Press.
Martin, R.N.D. and Stuart Brown (1988) *Discourse on Metaphysics and Related Writings*. New York: St. Martin's Press.
Massumi, Brian (2007) 'Potential Politics and the Primacy of Preemption', *Theory & Event* 10(2).
Meredith, Michael (2008) 'Never Enough (Transform, Repeat Ad Nausea)', pp. 7–9 in Tomoko Sakamoto, Albert Ferre and Michael Kubo (eds) *From Control to Design: Parametric/Algorithmic Architecture*. Barcelona: Actar-D.
Robinson, Abraham (1979) *Selected Papers of Abraham Robinson*, vol. 2: *Nonstandard Analysis and Philosophy*, edited by WAJ Luxemburg and S Körner. New Haven, CT: Yale University Press.
Roche, François (2009) 'I've Heard About... (A Flat, Fat, Growing Urban Experiment). Extracts of Neighbourhood Protocols', in Neil Leach (eds) *Digital Cities*, special issue, *Architectural Design* 79(4): 40.
Schumacher, Patrick (2009) 'Parametricism: A New Global Style for Architecture and Urban Design', in Neil Leach (ed.) *Digital Cities*, special issue, *Architectural Design* 79(4): 14–24.
Senagala, Mahesh (2001) 'Speed and Relativity: Toward Time-like Architecture', 89th ACSA Annual Meeting Proceedings, Baltimore. URL (consulted October 2010): www.mahesh.org.
Spuybroek, Lars (2004) *Nox: Machinic Architecture*. London: Thames and Hudson.
Whitehead, Alfred N. (1978) *Process and Reality*. New York: The Free Press.
Whitehead, Alfred N. (1997) *Science and the Modern World*. New York: The Free Press.

Luciana Parisi is the Convener of the Interactive Media MA, Critical Theory and Practice, at the Centre for Cultural Studies, Goldsmiths, University of London. In 2004 she published *Abstract Sex: Philosophy, Biotechnology and the Mutations of Desire* (Continuum Press). She has now finished a monograph entitled *Contagious Architecture: For an Aesthetic Computation of Space* (MIT, forthcoming 2013).